Array and Network Methods

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ABSTRACT

This document summarizes some key principles of two categories of methods for processing waveform data from groups of sensors: array methods, and network methods.

Arrays and Networks

Seismologists have developed a variety of ways of processing seismic data. As a conceptual model, these methods can be categorized into 'single station', 'array', 'sub-network', and 'network' methods (see Figure 1). These different categories differ in terms of whether/how multiple spatially-separated measurements are used, and the subsequent assumptions made about signals and noise.

The two most common methods for processing waveform data across groups of stations are array and network methods[†]. We define 'array' methods as the class of methods that assume some form of waveform coherence across different spatially-separated sensors. In contrast, 'network' methods do not make this assumption, and typically treat waveforms at spatially-separated sensors as uncorrelated. A distinction between 'array' and 'network' methods thus depends on the spacing between array elements, and the wavelength of the signals.

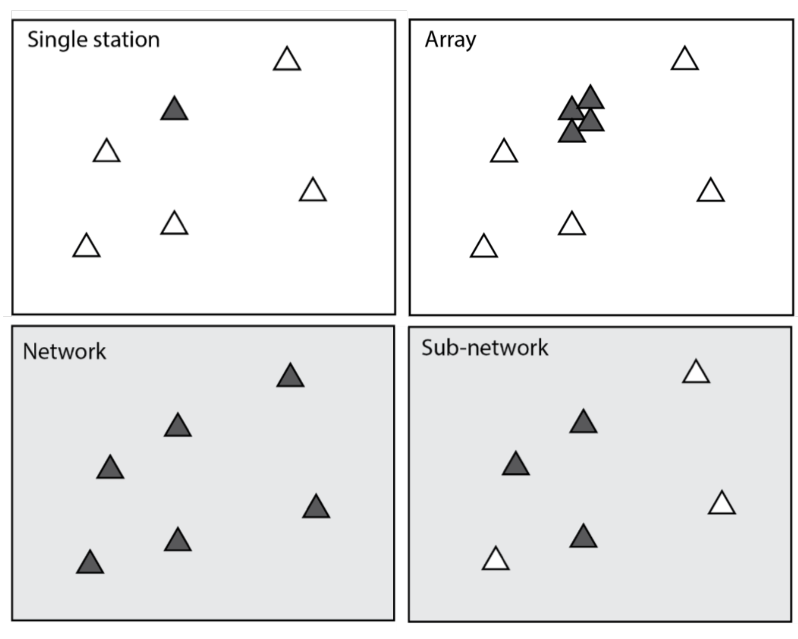

Figure 1. Conceptual diagram illustrating single stations, arrays, sub-networks, and networks.

Arrays

Arrays have played a key part of global seismology since the 1960's, and have played a key role in many applications of seismology. As shown in Figure 2, there are a variety of designs of seismic arrays, illustrating that there is no one-size-fits-all

[†]Methods that process waveform data at single-stations and associate parameters across a network (such as seismic phase arrival time measurements) are not considered 'network methods' in this paper, but are considered 'single-station' methods since waveform data are treated on a single-station basis.

solution for all applications. The array design depends on the phase of interest (e.g., teleseismic versus regional body waves), the Earth structure, land access, and the differing opinions of scientists about what constitutes a good array design! They are also a critical tool in processing infrasound data (where they are similarly varied).

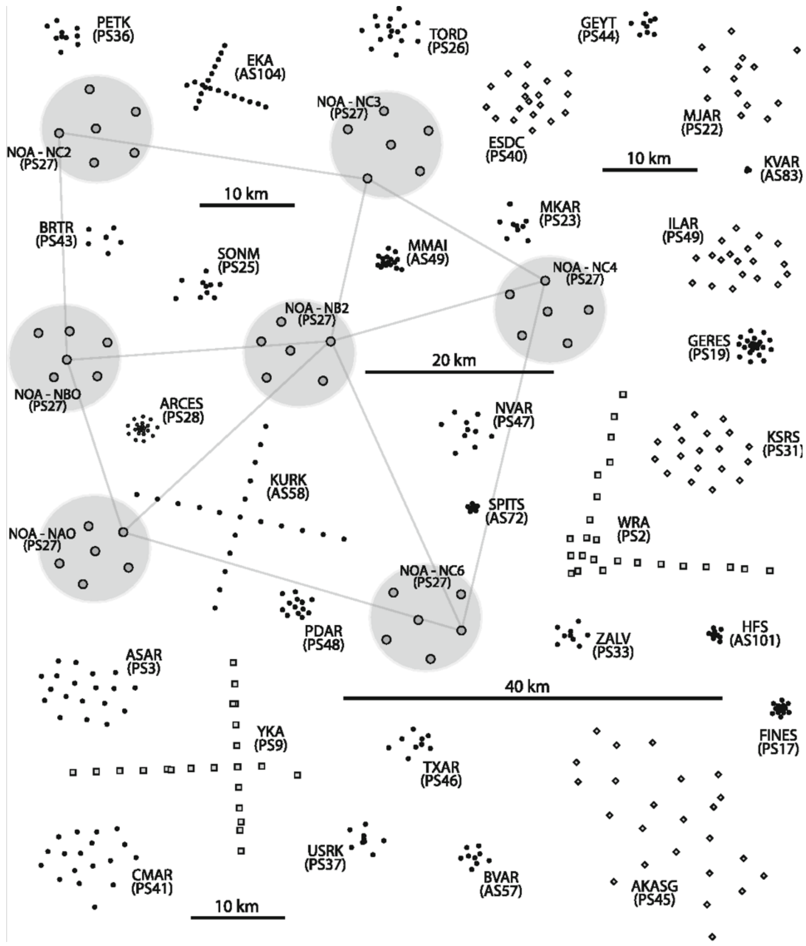

Figure 2. A map showing seismic arrays in the International Monitoring System (from Gibbons et al., 2013)

Plane Waves and Slowness

Array processing is typically based on the assumption of a plane wave arriving at an array. For seismic phases, which typically arrive at an angle θ from the vertical, a 2D cross-section showing the geometry in the vertical plane is shown in Figure 3.

In the time, Δt, that the wave propagates a distance, Δs, the apparent distance the wave has propagated along the surface is Δx. Following the geometry shown, the inclination angle and distances are related through:

$$\sin\theta = \frac{\Delta s}{\Delta x}. \tag{1}$$

Since $\Delta s = v_0 \Delta t$, where v_0 is the medium velocity, we have:

$$v_0 \Delta t = \Delta x \sin\theta, \tag{2}$$

and we define the horizontal slowness as:

$$\frac{\Delta t}{\Delta x} = \frac{\sin\theta}{v_0} = u_0 \sin\theta = u_{\text{hor}} \tag{3}$$

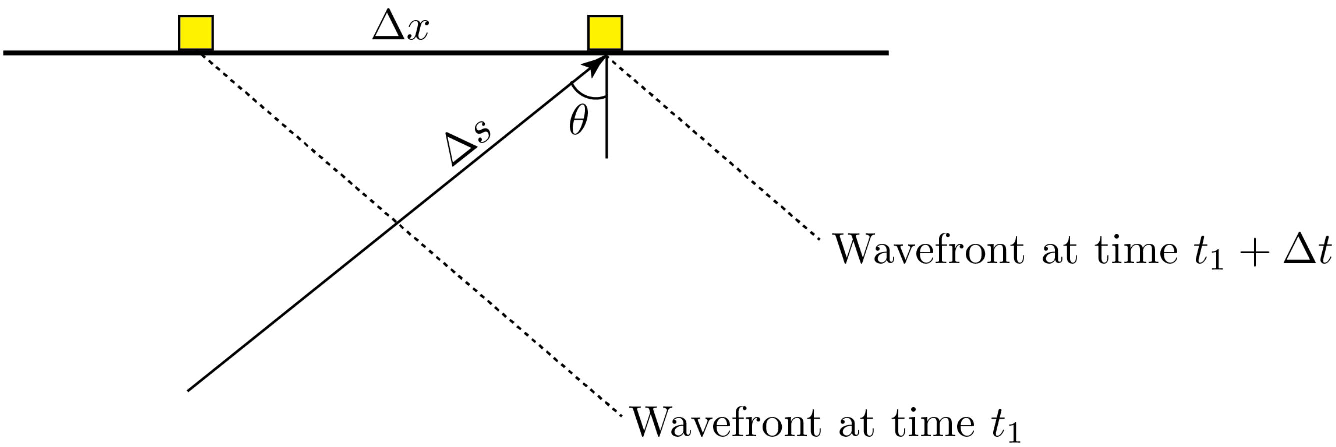

Figure 3. The geometry of a wave arriving at two elements of an array in (x,z) coordinates. Modified from a description in Shearer (2009).

In this case, the medium slowness is simply the reciprocal of the medium velocity: $u_0 = \frac{1}{v_0}$. For array processing, it is conventional to work in terms of slowness, which does not tend towards infinity as the wave arrives at steeper angles.

The direction-of-arrival (DOA) of a planar wavefront recorded on a 2D array can be described by the slowness vector $\underline{u} = (u_x, u_y)$. The slowness vector can be expressed in terms of two components: the backazimuth, $\phi = \tan^{-1}\left(\frac{u_y}{u_x}\right)$, and the phase (or trace) velocity (the inverse of the horizontal slowness): $v_p = \frac{1}{||\underline{u}||} = \frac{1}{u_{\text{hor}}} = \left(\sqrt{u_x^2 + u_y^2}\right)^{-1}$. These parameters are represented in Figure 4. The origin corresponds to a vertically arriving wave, which has a slowness of 0.

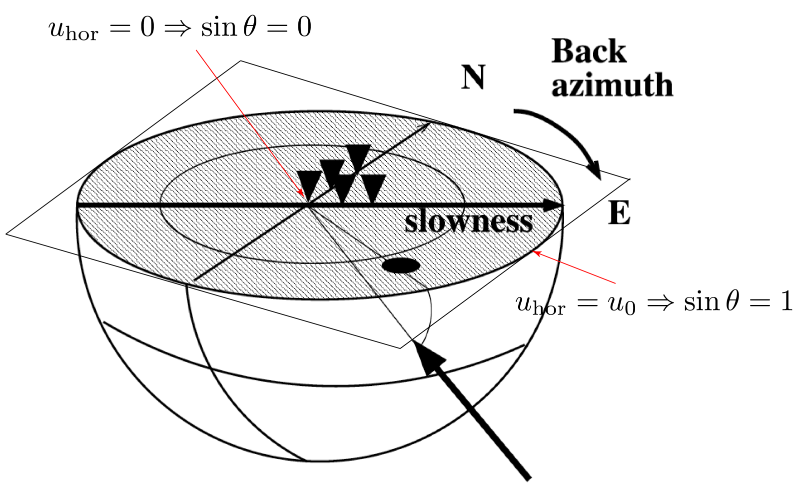

Figure 4. The geometry of a wave arriving at two elements of an array in 3D. An incident wave (represented by the solid black arrow) travels through a half sphere beneath the array. The distance of maximum power from the origin gives the horizontal slowness, while the angle from north gives the backazimuth. Modified from Rost and Thomas (2002).

Beamforming

If the backazimuth and phase velocity of a signal are known, the signals at each element can be time-shifted and summed to enhance the signal-to-noise ratio (the signal will constructively interfere, while noise will destructively interfere).

Given a reference location (typically the geographic center of the array, or an element near the center), the location of the i'th array element is given as $\underline{r}_i$. Thus, if the slowness vector is known, the time shift to that array element can be calculated from the dot product: $\tau_i = \underline{r}_i \cdot \underline{u} = x_i u_x + y_i u_y$. This process is called beamforming and is illustrated schematically in Figure 5.

If the observation at the i'th array element is written as $x_i(t)$, the beam for $\underline{u}$ is given by:

$$\bar{x}(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t+\tau_i) = \frac{1}{N}\sum_{i=1}^{N} x_i(t+\underline{r}_i \cdot \underline{u}) \tag{4}$$

Beamforming has been theoretically shown to improve the signal-to-noise ratio (SNR) of an array versus a single station by a factor of $\sqrt{N}$, assuming perfectly correlated signals and uncorrelated noise.

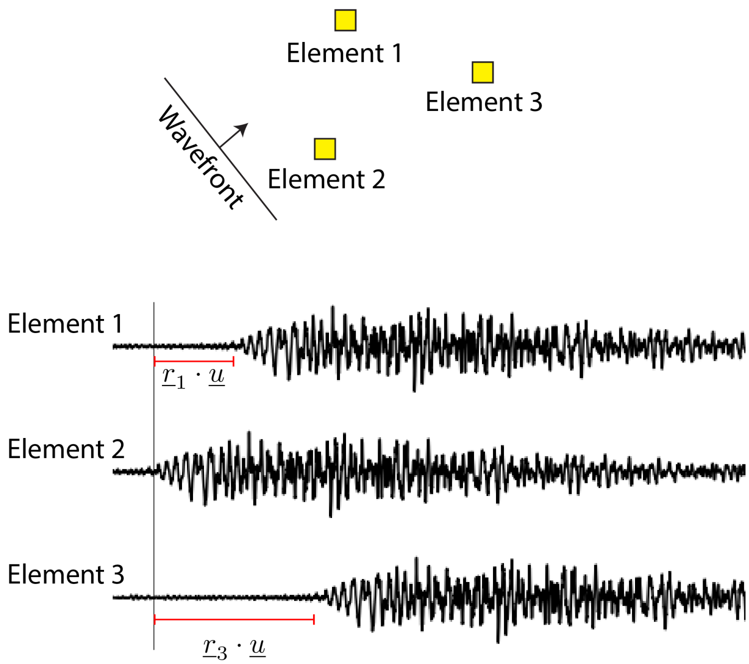

Figure 5. Beamforming involves time-shifting the traces at each element based on the known (or trial) slowness vector representing a planar wavefront. In this case, if Element 2 is the reference array location, time delays at Elements 1 and 3 can be calculated from $\underline{r}_1 \cdot \underline{u}$ and $\underline{r}_3 \cdot \underline{u}$.

Estimating the slowness vector

If the DOA of a signal is unknown, it can be estimated from the time shifts of the signals at the different array elements. This can be done in a few different ways that can be largely categorized as time-domain and frequency-domain methods. In the case of frequency-domain methods, the time shifts are handled through phase shifts of the different harmonic components of the waveform.

Least-squares inversion of pairwise time delays

A common method is to estimate inter-element delay times of signals, commonly through waveform cross-correlation. For a given pair of traces at elements j and k, the time delay, τ_{jk}, is found by finding the value that maximizes:

$$c_{jk}(\tau) = \int x_j(t)x_k(t-\tau)dt. \tag{5}$$

The resultant vector of M inter-element delay times, $\underline{\tau}$, which has dimension $M \times 1$, can be expressed in terms of the slowness vector, as:

$$\underline{\tau} = \underline{R}\underline{u} + \underline{\delta}. \tag{6}$$

In this equation, $\underline{R}$ has dimensions $M \times 2$ and contains the differences in coordinates for each pair of elements, $\underline{u} = (u_x, u_y)$, and $\underline{\delta}$ is the $M \times 1$ vector of residuals. $M = \frac{N(N-1)}{2}$ is the number of pairs of elements in an array.

The system of equations is usually solved by minimizing the sum of the squared residuals:

$$\underline{\hat{u}} = \underset{\underline{u}}{\text{minimize}} \sum_{i=1}^{M} |\delta_i(\underline{u})|^2 \tag{7}$$

where δ_i is the i'th component of the residual vector. This system of equations can be solved using the standard least-squares method:

$$\underline{u} = \left(\underline{R}\underline{R}^T\right)^{-1} \underline{R}^T \underline{\tau}. \tag{8}$$

Frequency-Wave Number Analysis

The least-squares method described above can only measure a single signal in a given window (and thus cannot resolve coincident arrivals from different directions). In contrast, the Frequency-Wave Number (FK) analysis can estimate the power distributed as a function of DOA, and thus can enable resolving multiple signals. The method works by defining a 'slowness grid' over trial values of $\underline{u} = (u_x, u_y)$ and estimating the power of the beamformed signal for each trial value of slowness. The slowness grid should cover a plane over slowness values that extend out at least to the medium slowness, u_0, as shown by the plane over the half sphere in Figure 4.

Recall that the beamformed signal for a trial value of $\underline{u} = (u_x, u_y)$ is calculated as:

$$\bar{x}(t) = \frac{1}{N} \sum_{i=1}^{N} x_i\left(t + \underline{r}_i \cdot \underline{u}\right) \tag{9}$$

Thus, the power in a time window with N_T samples is estimated as:

$$P(u_x, u_y) = \frac{1}{N_T} \sum_{j=1}^{N_T} \bar{x}(t)^2 = \frac{1}{N_T} \sum_{j=1}^{N_T} \left[\frac{1}{N} \sum_{i=1}^{N} x_i\left(t + \underline{r}_i \cdot \underline{u}\right)\right]^2 \tag{10}$$

To optimize computational efficiency, the calculation is performed in the frequency domain, over a frequency band where $f_1 \leq f \leq f_2$. The power can be expressed as:

$$P(u_x, u_y) = \frac{1}{N_T^2} \sum_{f=f_1}^{f_2} \left| \frac{1}{N} \sum_{i=1}^{N} X_i(f) e^{-i2\pi f \underline{r}_i \cdot \underline{u}} \right|^2 \tag{11}$$

where the normalization factor of $\frac{1}{N_T^2}$ follows from Parseval's theorem.

Sliding-window array processing

Sliding-window methods form the basis for processing data from an array when one is interested in detecting signals, or in exploring the evolution in DOA of a signal with time. An extension of this method includes a frequency axis, where sliding-window array processing may be applied in different frequency bands.

Assuming a single frequency band, the implementation of sliding-window array processing is explained in Figure 6. In sliding-window array processing, the DOA of a best-fitting plane wave is estimated in a moving time window using any suitable method (two of the most common methods are described in the previous section). Typically, successive time windows would overlap by some amount in order to improve the resolution of time-varying effects and signal onset times.

It is common to estimate a measure of the power of a signal associated with the DOA in each time window, as this can form the basis of a binary signal detection problem. A common approach is to normalize the power calculated on the beamformed signal to the ambient power from all other directions-of-arrival. This can be computed from the 'semblance', which is calculated as:

$$S = \frac{\sum_{t=t_1}^{t_1+\Delta t} \bar{x}(t)^2}{N \sum_{i=1}^{N} \sum_{t=t_1}^{t_1+\Delta t} x_i(t)^2}, \tag{12}$$

where $\bar{x}(t)$ is the beam, as defined previously. The semblance can be related to another detection statistic called the F-ratio through the following relation:

$$F = (N-1)\frac{S}{1-S}. \tag{13}$$

The F-ratio is often used as a detection statistic in signal detection problems because it has a known statistical distribution in the presence of uncorrelated White noise.

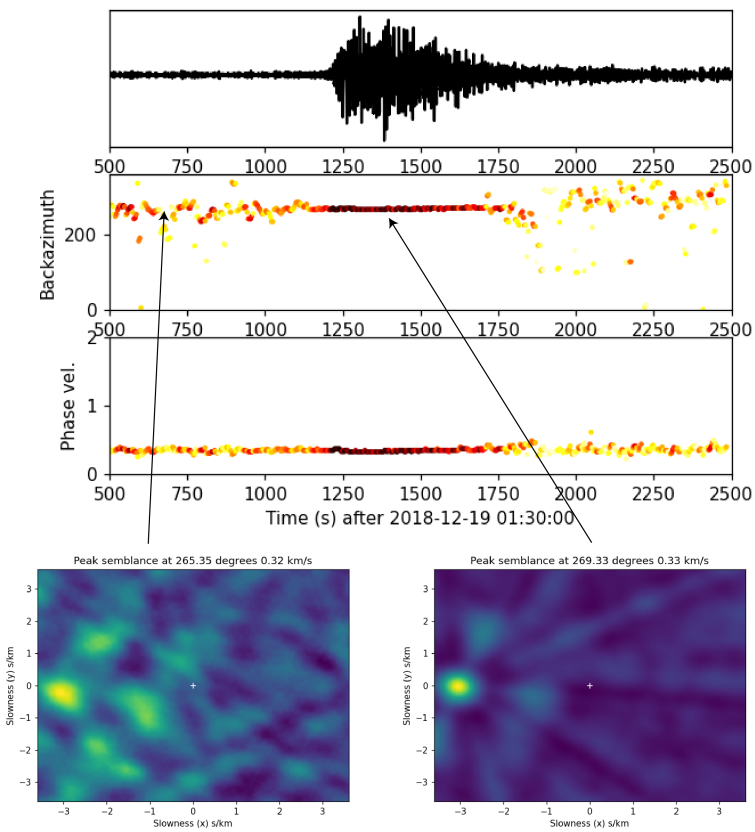

Figure 6. Example of sliding-window array processing for an infrasound signal from a large bolide recorded at the I53US infrasound array in Fairbanks, Alaska. The backazimuth and phase velocity estimates are color-coded by the semblance. The power as a function of the slowness vector, $\underline{u}$, is shown for two example time periods.

Networks

A network is a group of stations that can be processed collectively, but where the signals are not necessarily coherent at the stations. Network methods thus exploit the incoherent properties of waveforms, and typically remove the phase information in waveforms by transforming each waveform to a positive real scalar function. Example transforms include the envelope, ratio of short-term average to long-term average (STA/LTA), absolute value of the time series, or the derivative of the Kurtosis. The increasing use of dense networks of sensors in seismology (e.g., USArray, Large-N deployments) has led to the development of techniques that adapt classical array methods for processing data on dense networks. These can be thought of as 'sub-network' methods (Figure 1), as they often process waveform data with subsets of stations before subsequently stacking across a network.

Some common transforms

The STA/LTA transform is based on power, analogous to the array methods described above, but considers the ratio of power in a short-term to a long-term window at a single station:

$$a = \frac{\frac{1}{N_s}\sum_{i=1}^{N_s} x_i^2(t)}{\frac{1}{N_l}\sum_{i=1}^{N_l} x_i^2(t)} \tag{14}$$

where x is the filtered seismic data, N_s is the number of samples in the short-term window, and N_l is the number of samples in the long-term window. The short-term window typically leads the long-term window as shown in Figure 7.

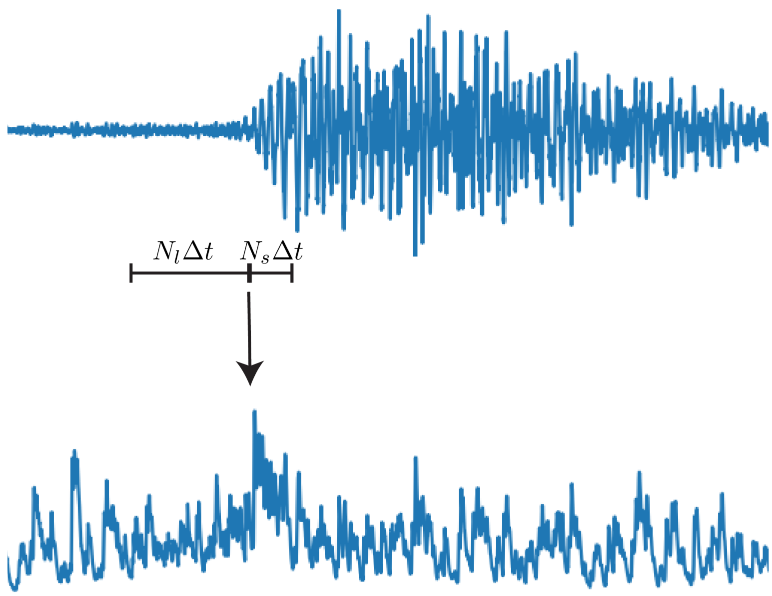

Figure 7. STA/LTA is usually implemented with a leading short-time window of duration $N_s\Delta t$ (where Δt is the sampling rate) and trailing long-time window of duration $N_l\Delta t$.

Backprojection

A class of network methods for event detection and location 'backproject' transformed waveform data to a range of possible event hypotheses in order to evaluate their fit. To do so requires a model that can predict the travel-time $t_{\underline{r}i}$ from a given trial location $\underline{r} = (\lambda, \phi, z)$ to the i'th receiver:

$$P(\tau, \underline{r}) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{m=0}^{M-1} a(\tau + t_{\underline{r}i} + m\delta t)\right] \tag{15}$$

where: a is the STA/LTA transformed time series, N is the number of stations, τ is a trial origin time M is the number of points within a time window that starts at the predicted arrival time, and δt is the sampling interval of the data. A typical implementation of this equation requires the specification of a 4D grid of event hypotheses (represented by $\tau, \underline{r}$). Such an implementation is illustrated for 3D (without depth) in Figure 8. By computing $P(\tau, \underline{r})$ over a range of trial event hypotheses, the event detection and location problem is reduced to finding spatiotemporal peaks in P.

Mesh-of-Arrays

The basic principles of time-domain array processing methods can be applied to groups of stations where the signals are not coherent by transforming the data to remove phase, as described above. Methods such as beamforming, and least-squares inversion of the slowness vector, can be applied to such data. One example of such a method, which was developed for processing USArray data, is the AELUMA method described by de Groot-Hedlin and Hedlin (2015). AELUMA breaks down the network into triads of stations, which are processed incoherently by computing envelopes. Each triad (3-station group)

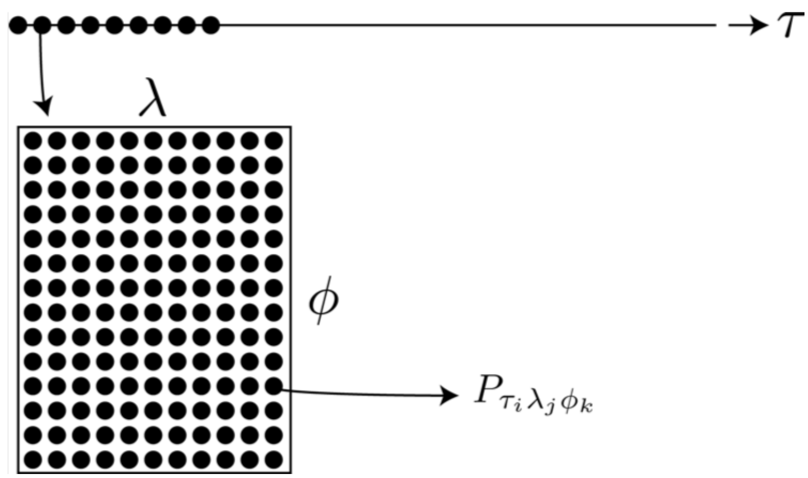

Figure 8. Backprojection methods typically require the specification of a set of possible event locations and origin times (a single event hypothesis is represented in this figure by an origin time and only two spatial dimensions: $\tau_i, \lambda_j, \phi_k$). Observed data are then backprojected for all possible event hypotheses to estimate some real-valued function P that measures the fit to that hypothesis.

is processed using cross-correlation to estimate delay times between the three station pairs within the triad. If a given triad contains stations i, j, and k, the detection statistic used is:

$$\tau_c = \tau_{ij} + \tau_{jk} + \tau_{ki}. \tag{16}$$

The statistic τ_c is referred to as the 'consistency' † and is equal to zero if the delay times between the three pairs are consistent with a plane wave:

$$\tau_{ij} + \tau_{jk} + \tau_{ki} = (t_i - t_j) + (t_j - t_k) + (t_k - t_i) = 0. \tag{17}$$

If the consistency is below a threshold, the DOA is estimated across the triad and events are detected when the DOA's across multiple triads in a network are consistent with a common event.

Local Similarity

A network method that was recently developed for Large-N seismic networks is the Local Similarity method (Li et al., 2018). In common with the Mesh-of-Arrays approach, Local Similarity breaks a network down into subsets of sensors and then stacks the results. However, Local Similarity can be considered a hybrid between array and network methods because it does not operate on a positive value transform of the data, but retains the phase information in each waveform. Each station (the 'master' station) is grouped with its closest K stations (the 'neighbors'), and the 'local similarity' is computed as the mean of the normalized cross-correlations, c_{ij} between the master and neighbors:

$$s_i(t) = \frac{1}{K} \sum_{j=1}^{K} c_{ij}(t). \tag{18}$$

The resulting values of $S_i(t)$ for all N stations in a network are subsequently stacked by simply adding together the individual local similarity functions, to give the network local similarity:

$$S(t) = \sum_{i=1}^{N} s_i(t). \tag{19}$$

† The same statistic forms the basis of an array processing method that is widely used in the infrasound community: the PMCC method (Cansi, 1991)